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## Free Streaming Length and Constraints on Dark Matter:

After dark matter particles completely decouple from the plasma they move freely. These particles have at the time of decoupling a velocity distribution that is given by thermal value. However, they move randomly as their velocities do not have the same phase.

Between the time of decoupling and the radiation-matter equality free streaming is the dominant effect. Note that the universe is not exactly homogeneous and there are tiny density fluctuations that eventually act as seeds for structure formation. As we will see later on, the inhomogeneities of dark matter undergo growth because of gravitational clumping only after matter dominance. Moreover, inhomogeneities ~~of~~ baryons can only grow after the recombination.

However, the growth of fluctuation after the matter radiation equality depends on the amplitude of fluctuations at that time. Random motion of dark matter particles between the decoupling time ($t_{dec}$) and the time of matter radiation equality ($t_{EQ}$) dampens inhomogeneities in this interval. As mentioned, this is due to particles having velocities with random phases, and is similar to spreading of a concentration of smoke because of diffusion. The wash out of fluctuations as a result of free streaming is called Landau damping (or collisionless damping). It affects all fluctuations whose comoving wavelength is smaller than or comparable to the comoving distance travelled during the free streaming.

We expect the effect to be more important in the case that dark matter particles are in the relativistic regime at the time of decoupling. The reason being that they have a larger velocity in this case, and hence travel a larger distance. We therefore focus on this case and calculate the free streaming length, denoted by $\lambda_{FS}$.

We have:

$$\lambda_{FS} \simeq \int_{t_{dec}}^{t_{NR}} \frac{dt'}{a(t')} + \int_{t_{NR}}^{t_{EQ}} \frac{a_{NR}}{a^2_{(t')}} dt'$$

The first integral represents the comoving distance (not the scale factor "a" in the denominator) travelled between decoupling and the time the particle goes into the non-relativistic regime. Within this period one can use the approximation that the physical velocity is $\sim 1$ (in natural units).

The second integral is the comoving distance travelled in the ~~non~~-relativistic regime. The physical velocity is redshifted $\propto a^{-1}$ in this regime, hence $a^2$ in the deominator of the integrand.

After using the fact that the universe is in a radiation-dominated phase for $t < t_{EQ}$, where $a(t) \propto t^{\frac{1}{2}}$, we find:

$$\lambda_{FS} \simeq \frac{2 t_{NR}}{a_{NR}} + \frac{t_{NR}}{a_{NR}} \ln\left(\frac{t_{EQ}}{t_{NR}}\right)$$

At $t_{dec}$ we have $t_{dec} \propto T_{dec}^{-2}$, while at $t_{NR}$ we have $t_{NR} \propto T_{NR}^{-2}$. Here $T_{dec}$ and $T_{NR}$ denote the plasma temperature at $t_{dec}$ and $t_{NR}$ respectively.

Temperature of the dark matter particle $X$, denoted by $T_X$, is in general different from the plasma temperature exactly because of its decoupling from the plasma. At $t_{NR}$ we have $T_X \simeq \frac{m_X}{3}$ (factor of 3 is because of the fact that average kinetic energy of a relativistic particle at temperature T is 3T). We can therefore find:

$$\boxed{\begin{aligned} t_{NR} &\simeq 1.2 \times 10^{7} \left(\frac{1\,keV}{m_X}\right)^2 \left(\frac{T_X}{T}\right)^2 \sec \\ a_{NR} &\simeq 7.1 \times 10^{-7} \left(\frac{1\,keV}{m_X}\right) \left(\frac{T_X}{T}\right) \end{aligned}}$$

Note that $\frac{T_X}{T}$ is a constant, and hence we can use the value anytime after the decoupling of $X$ from the plasma.

Also, we find:

$$\boxed{\left(\frac{t_{NR}}{t_{EQ}}\right)^{-1} \simeq \left[\left(\frac{m_X}{1.0\,eV}\right)\left(\frac{T}{T_X}\right)\right]^2}$$

Recall that $t_{EQ} \sim 100{,}000$ yr and $T_{EQ} \sim 1.5$ eV.

Putting everything together, we reach at:

$$\boxed{\lambda_{FS} \simeq 0.2\ Mpc \left(\frac{1\,keV}{m_X}\right)\left(\frac{T_X}{T}\right)\left[\ln\left(\frac{t_{EQ}}{t_{NR}}\right) + 2\right]} \quad (*)$$

We see that the smaller $m_X$ is, the larger $\lambda_{FS}$ will be. This can be understood intuitively: a lighter particle will be in the relativistic regime for a longer time, and hence travels a longer distance.

Clearly, free streaming affects inhomogeneities with a smaller wavelength more. The smallest cosmological structures that we see today, galaxies, could not have formed if free streaming had erased fluctuation with a comoving wavelength around the comoving size of galaxies.

Now, let us use this to constrain the mass of light neutrinos as dark matter. As we saw before, $\frac{T_\nu}{T} = \left(\frac{4}{11}\right)^{\frac{1}{3}}$ in this case.

We therefore find:

$$\tilde{\lambda}_{FS} \sim 20\,\mathrm{Mpc}\left(\frac{30\,\mathrm{eV}}{m_\nu}\right)$$

Requiring that $\tilde{\lambda}_{FS} \ll 0.5\,\mathrm{Mpc}$, the neutrino mass is constrained to be $m_\nu \gg 1\,\mathrm{keV}$. However, neutrinos this massive exceed the Cowsik-McClleland bound badly and cannot be dark matter candidates because their contribution to the energy density of the universe at the present time is several times that of dark matter.

This, irrespective of experimental bounds on $m_\nu$, rules out light neutrinos as the dominant component of dark matter.

Looking at the expression in (a) shows that non erasure of inhomogeneities due to free streaming sets a lower bound on $m_\chi$. Therefore a light dark matter candidate cannot be arbitrarily light. In other words, it cannot be "hot" (ultrarelativistic). Rather it should be "warm" (relativistic) to be a viable candidate. Currently, observations from CMB and large scale structure prefer "cold" dark matter (non-relativistic at the time of decoupling) as the dominant component. A "warm" component, however, is still viable and may cure some of the problems of cold dark matter at small scales.